A Dynamic Space-Time Regression Model of Plot-Level Forest Biomass Data in Maine, USA

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Motivation

- The Forest Inventory and Analysis (FIA) program of the US Forest Service has systematically collected data from forest inventory plots across the US for more than 20 years.
- These data include estimates of forest biomass, which is an important metric for monitoring forest carbon and climate change.
- FIA plots are measured on a rotating basis every 5 years, creating a spatio-temporal dataset with both observed and unobserved data.

Motivation

Plot	1998	1999	2000	2001	2002	2003	2004	2005
1					20.1			
2	30.8					32.4		
3				19.5				
4		36.2					35.0	
5			29.4					28.3

Objectives

- Develop a dynamic space-time regression model that:
  - estimates plot-level forest biomass data for FIA plots in Maine, USA with associated uncertainty quantification for all plot locations and years,
  - incorporates wall-to-wall covariates (i.e. remotely sensed data),
  - dynamically characterizes the spatial dependence between FIA biomass measurements over time,
  - utilizes an efficient low-rank approximation for the spatial process (i.e. Predictive Process).

Model Notation

- let the data be observed at locations $\mathcal{S} = \{s_1, s_2, \ldots, s_n\}$ and at time points $1, 2, \ldots, T$
- we assume that there is missing/unobserved data
- $y_t(s)$ denotes the plot-level biomass at location s and time t
- $z(s)$ is the $q \times 1$ time independent fixed effect covariate (i.e. average climate, etc.) observed at location s, γ is the corresponding $q \times 1$ coefficient vector
- $x_t(s)$ is the $p \times 1$ time dependent fixed effect covariate (i.e. NDVI, etc.) observed at location s at time t, β_t is the corresponding $p \times 1$ coefficient vector
- also let $I_t(s)$ denote the indicator variable denoting if the data is observed at location s at time t

Model Definition

The model is defined as

$$y_t(s) = z(s)'\gamma + x_t(s)'\beta_t + u_t(s) + \epsilon_t(s), \epsilon_t(s) \overset{\text{iid}}{\sim} N(0, \tau_t^2) \text{ if } I_t(s) = 1$$

$$\beta_t = \beta_{t-1} + \eta_t, \eta_t \overset{\text{iid}}{\sim} N(0, \Sigma_\eta), \beta_0 \sim N(m_0, \Sigma_0), \gamma \sim N(m, \Sigma)$$

$$u_t(s) = u_{t-1}(s) + w_t(s), w_t(s) \overset{\text{iid}}{\sim} GP(0, C_t(\cdot, \theta_t))$$

$$u_0(s) = 0, \text{ for } t = 1, 2, \ldots, T \text{ and generic location } s$$

with hyper priors: $\tau_t^2 \overset{\text{iid}}{\sim} IG(a_\tau, b_\tau), \Sigma_\eta \sim IW(r_\eta, \Omega_\eta), \theta_t \overset{\text{iid}}{\sim} p_t(\theta_t)$

Predictive Process and NNGP

- the Guassian process used to model the spatial random effects can be replaced by a lower dimensional approximation to ease computation
- the predictive process model (motivated from kriging) approximates the parent (full) process through a linear combination of 'knots', forming a low rank covariance matrix
- the nearest neighbor Gaussian process (NNGP) model generates a sparse covariance matrix through a 'screening effect' by considering m nearest neighbors

Model Implementation

We assume an exponential spatial covariance function of the form

$$C_t(s_1, s_2; \theta_t) = \sigma_t^2 exp(-\phi_t ||s_1 - s_2||)$$

where $\theta_t = \{\sigma_t^2, \phi_t\}$ with spatial variance σ_t^2 and spatial decay ϕ_t

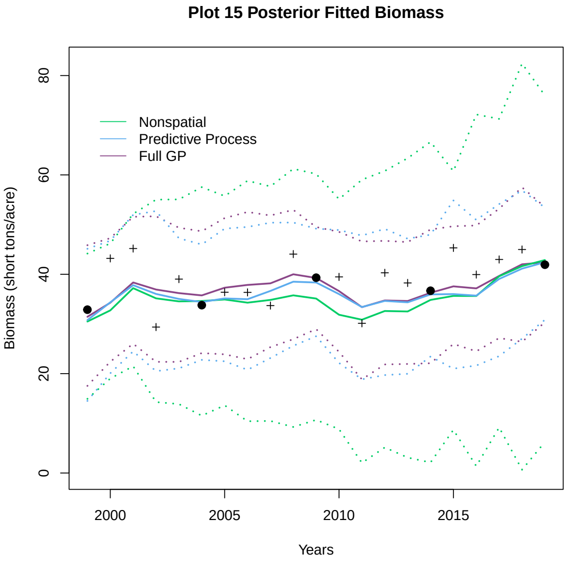
Model Implementation

- these prior specifications lead to a well-defined Bayesian hierarchical model
- Bayesian inference proceeds through a Gibbs sampling algorithm with random-walk Metropolis steps for spatial covariance parameters
- full GP and predictive process models are called from the `spBayes` R package
- samplers are written in `C++` using `LAPACK` and `BLAS` linear algebra subroutines

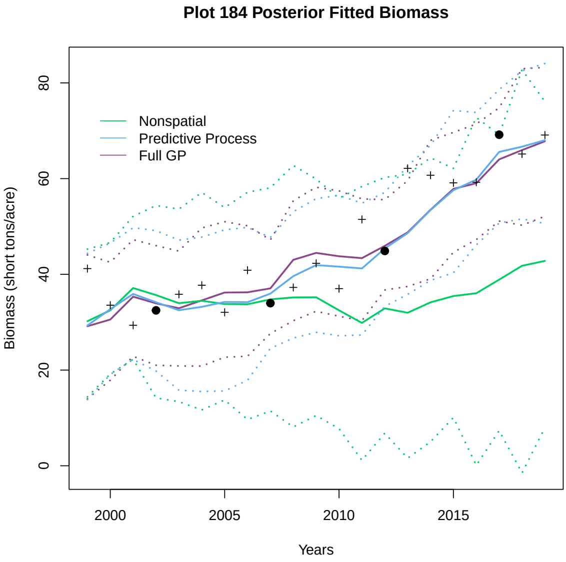
Simulation

- single temporally-varying intercept
- $T = 21$ years
- $n = 400$ locations (single county in Maine, USA)
- 6,795 of 8,400 plot-year combinations (80.9%) are unobserved
- consider nonspatial, predictive process (25 knots), and full GP models
- parameter estimates summarized from 2,500 post burn-in posterior samples

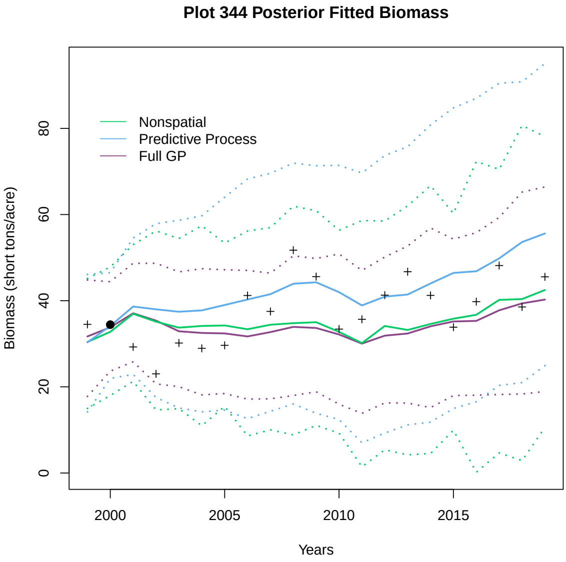
Preliminary Results



Preliminary Results



Preliminary Results



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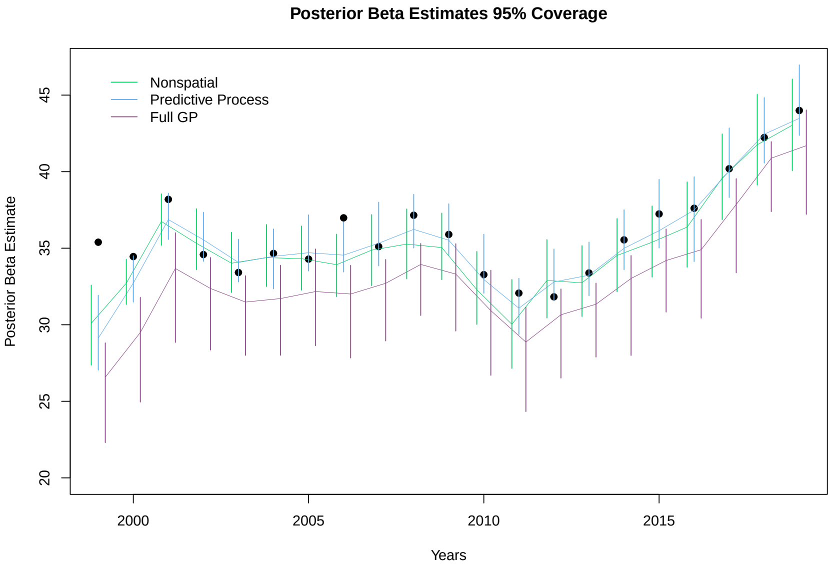

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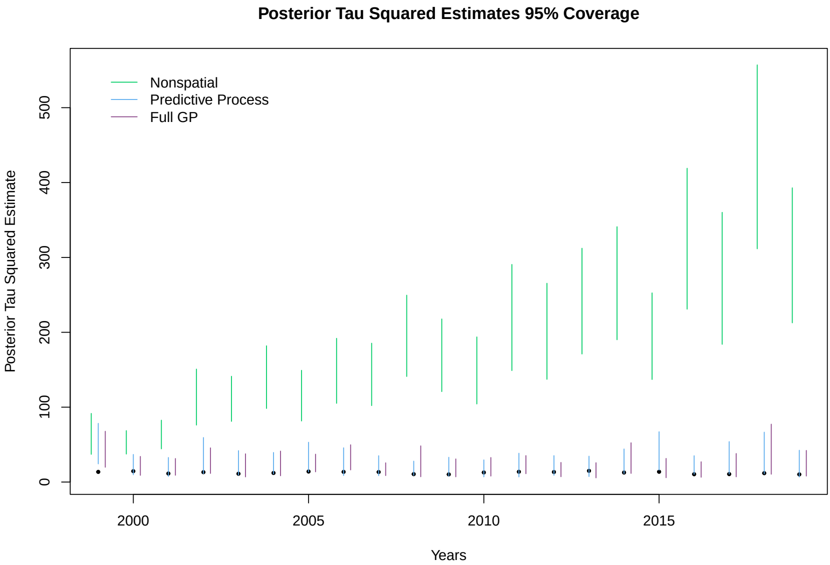

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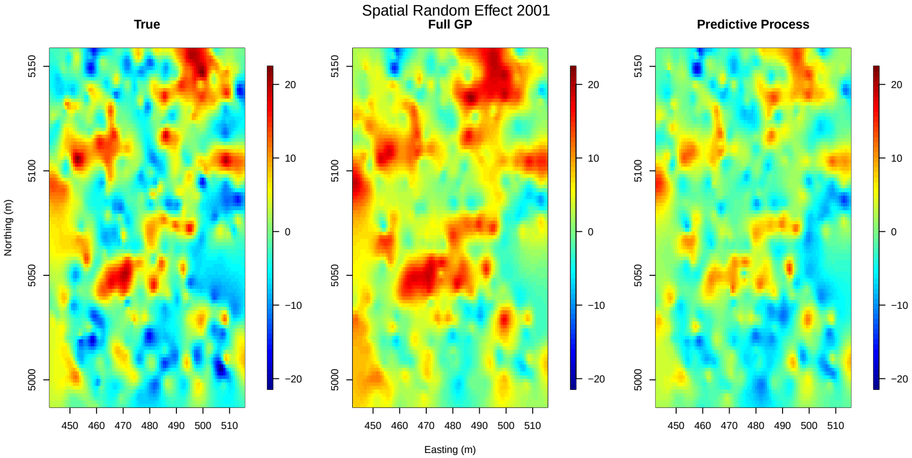

Application Data

- FIA plot-level biomass measurements
  - collected from 1999 - 2019
  - $n = 2{,}713$ plot locations
  - 10,833 total observations
- Landsat Normalized Difference Vegetation Index (NDVI)
  - annual composite images from 1999 - 2019
  - quantifies vegetation greenness and represents vegetation density
- LandTrendr
  - spectral-temporal segmentation algorithm for detecting change from satellite imagery (Landsat)
  - identifies and quantifies forest disturbance from annual image composites

Next Steps

- make predictions of forest biomass at arbitrary locations and times within the spatio-temporal domain
- implement NNGP for spatial random effect

References

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