A Multivariate Spatio-Temporal Fay-Herriot Model for Forest Carbon Pools Across the Contiguous US

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Motivation

- The United Nations require annual reporting of greenhouse gas emissions from five sectors:
  - Energy
  - Industry
  - Agriculture
  - **Forestry**
  - Waste
- The quality of these reports relies on the data and methods used to make estimates.

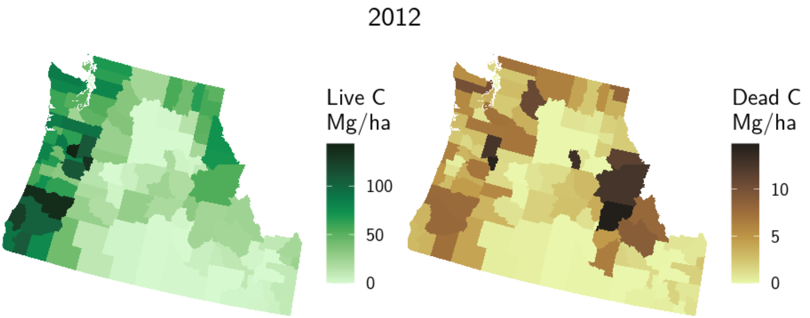
Source: Eggleston et al. (2006)

Motivation

- The Forest Inventory and Analysis (FIA) program of the US Forest Service measures forest carbon at inventory plots for different carbon "pools".
    - Live trees
    - Dead trees
    - Leaf litter
    - Soil
- Interested in status, trend, and change estimates at fine spatial and temporal scales (i.e., annual county-level estimates).

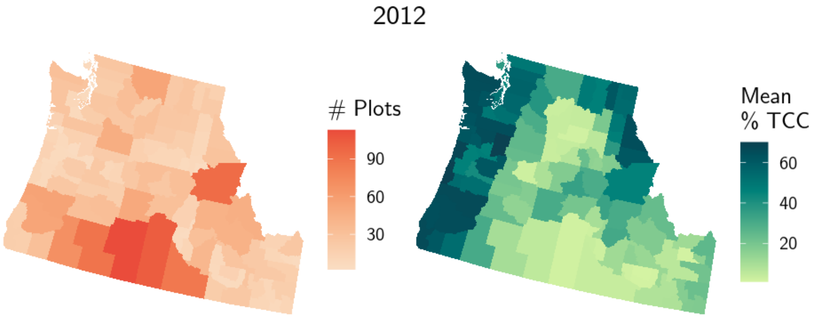
Data

- Carbon density measurements for each pool are collected on a rotating basis from inventory plots.
- Direct estimates are calculated at the county level for years 2008 – 2021.
- Due to sparseness of inventory plots and the small areas of interest, direct estimates may be missing for some counties/years.



Data

- Uncertainty in direct estimates from few inventory plots and/or measurement variation within a given county.
- Auxiliary data such as remotely sensed tree canopy cover (TCC) may be informative as a covariate.



Objectives

- Develop a dynamic, non-stationary, multivariate spatio-temporal model that:
    - accommodates spatial and temporal dependence *between* pools,
    - acknowledges dependence *among* pools, while allowing non-stationary correlations,
    - incorporate uncertainty stemming from direct estimates due to sample size and/or variability in measurements,
    - leverages available spatially and/or spatio-temporally varying predictors
    - provides improved carbon density estimates for each pool in all counties/years.

Direct Estimates

Let $y_{m,i,j,t}$ be the measured carbon density for the m^{th} carbon pool from the i^{th} plot in county j at time t, with $m = 1, \ldots, M$, $i = 1, \ldots, n_{m,j,t}$, $j = 1, \ldots, J$, and $t = 1, \ldots, T$.

The direct estimate of the mean carbon density of pool m in county j at time t is

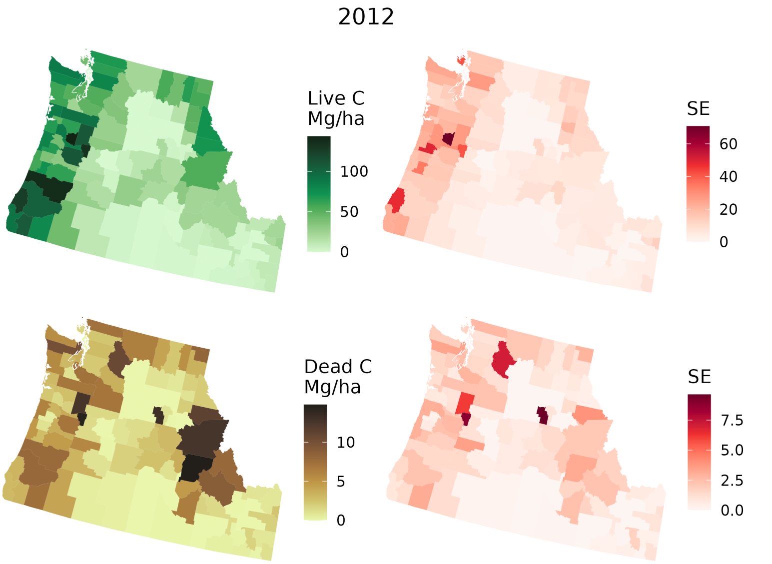
$$\hat{\mu}_{m,j,t} = \frac{1}{n_{m,j,t}} \sum_{i=1}^{n_{m,j,t}} y_{m,i,j,t} \tag{1}$$

with associated variance

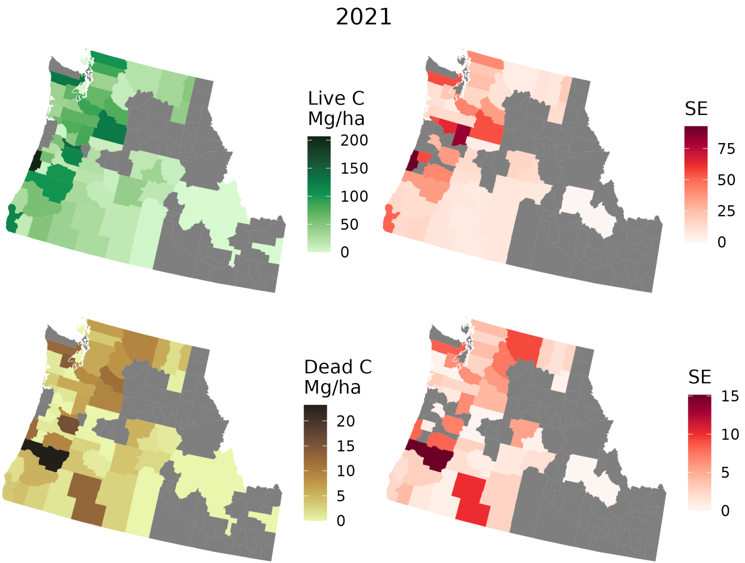
$$\hat{\sigma}^2_{m,j,t} = \frac{1}{n_{m,j,t}(n_{m,j,t} - 1)} \sum_{i=1}^{n_{m,j,t}} (y_{m,i,j,t} - \hat{\mu}_{m,j,t})^2 \tag{2}$$

Missingness occurs when $n_{m,j,t} = 0$, $n_{m,j,t} = 1$, or all measurements with a county/year are equal.

Direct Estimates



Direct Estimates



Proposed Model

For county j at time t, we are interested in the length M vector of latent means $\boldsymbol{\mu}_{j,t} = (\mu_{1,j,t}, \ldots, \mu_{M,j,t})^\top$.

We leverage direct estimates $\hat{\boldsymbol{\mu}}_{j,t} = (\hat{\mu}_{1,j,t}, \ldots, \hat{\mu}_{M,j,t})^\top$ and $\hat{\boldsymbol{\sigma}}^2_{j,t} = (\hat{\sigma}^2_{1,j,t}, \ldots, \hat{\sigma}^2_{M,j,t})^\top$, along with auxiliary data available for each county and time, $\mathbf{X}_{j,t}$.

The proposed Fay–Herriot (FH) model is

$$\hat{\boldsymbol{\mu}}_{j,t} = \boldsymbol{\mu}_{j,t} + \boldsymbol{\delta}_{j,t} \tag{3}$$

$$\boldsymbol{\mu}_{j,t} = \mathbf{X}_{j,t}\boldsymbol{\beta}_j + \mathbf{u}_{j,t} + \boldsymbol{\varepsilon}_{j,t} \tag{4}$$

Proposed Model

Model Components:

$$\hat{\boldsymbol{\mu}}_{j,t} = \boldsymbol{\mu}_{j,t} + \boldsymbol{\delta}_{j,t} \tag{3}$$

$$\boldsymbol{\mu}_{j,t} = \mathbf{X}_{j,t}\boldsymbol{\beta}_j + \mathbf{u}_{j,t} + \boldsymbol{\varepsilon}_{j,t} \tag{4}$$

- $\boldsymbol{\delta}_{j,t} \overset{\text{ind}}{\sim} MVN(\mathbf{0}, \boldsymbol{\Sigma}_{\delta,j,t})$ Direct estimate error term.
- $\mathbf{X}_{j,t}$ $M \times MP$ block diagonal matrix of P many predictor variables.
- $\boldsymbol{\beta}_j$ length MP vector of county-varying regression coefficients.
- $\mathbf{u}_{j,t}$ length M vector of county- and time-varying intercepts.
- $\boldsymbol{\varepsilon}_{j,t} \overset{\text{iid}}{\sim} MVN(\mathbf{0}, \boldsymbol{\Sigma}_{\varepsilon})$ Latent error term.

Spatio-temporal intercept

For $t = 1, \ldots, T$,

$$\mathbf{u}_{j,t} = \mathbf{u}_{j,t-1} + \mathbf{w}_{j,t}, \quad (\mathbf{u}_{j,0} \equiv 0) \tag{5}$$

$$\mathbf{w}_{j,t} = \mathbf{A}\mathbf{v}_{j,t} \tag{6}$$

$\mathbf{A}$ is the cholesky square root of the spatial random effects cross covariance matrix.
Elements of $\mathbf{v}_{j,t} = (v_{1,j,t}, \ldots, v_{M,j,t})^\top$ are modeled as

$$\begin{pmatrix} v_{m,1,t} \\ \vdots \\ v_{m,J,t} \end{pmatrix} \sim MVN\left(\mathbf{0}, \mathbf{Q}(\rho_{v,m,t})^{-1}\right), \quad m = 1, \ldots, M \tag{7}$$

where $\mathbf{Q}(\rho_{v,m,t})^{-1}$ is a CAR correlation matrix with spatial correlation parameter $\rho_{v,m,t}$ (Besag, 1974).

Spatially-varying coefficient

We have $\boldsymbol{\beta}_j = (\boldsymbol{\beta}_{1,1,j}, \ldots, \boldsymbol{\beta}_{M,P,j})^\top$.

The elements of $\boldsymbol{\beta}_j$ are again modeled using the CAR structure

$$\begin{pmatrix} \beta_{m,p,1} \\ \vdots \\ \beta_{m,p,J} \end{pmatrix} \sim MVN\left(\mathbf{0}, \tau^2_{\beta,m,p}\mathbf{Q}(\rho_{\beta,m,p})^{-1}\right), \quad \begin{matrix} m = 1, \ldots, M \\ p = 1, \ldots, P \end{matrix} \tag{8}$$

where $\tau^2_{\beta,m,p}\mathbf{Q}(\rho_{\beta,m,p})^{-1}$ is the CAR covariance matrix with scalar variance $\tau^2_{\beta,m,p}$ and spatial correlation parameter $\rho_{\beta,m,p}$.

Direct estimate variance

We consider $\mathbf{\Sigma}_{\delta,j,t} = \text{diag}\left(\sigma^2_{1,j,t}, \ldots, \sigma^2_{M,j,t}\right)$. where each $\sigma^2_{m,j,t}$ is given an

Inverse Gamma prior of the form

$$\sigma^2_{m,j,t} \sim IG\left(\frac{n_{m,j,t}}{2}, \frac{(n_{m,j,t}-1)\hat{\sigma}^2_{m,j,t}}{2}\right) \tag{9}$$

for $m = 1, \ldots, M$.

Proposed Model

Latent error:
A basic specification assumes $\mathbf{\Sigma}_\varepsilon = \text{diag}(\sigma^2_{\varepsilon,1}, \ldots, \sigma^2_{\varepsilon,M})$, with each $\sigma^2_{\varepsilon,m}$ following an Inverse Gamma prior.

Priors:
Finally, we specify vague Inverse Gamma priors for variance terms τ^2's, Uniform$(0, 1)$ priors on spatial dependence parameters ρ's, and an Inverse Wishart prior on $\mathbf{AA}^\top$.

Posterior samples are generated using Gibbs sampling for parameters with explicit full conditional distributions, and Metropolis steps for all other parameters.

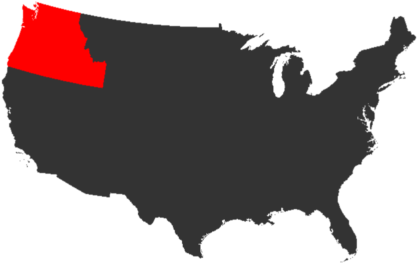
Preliminary Results Setting

- **Data**: Live and dead tree carbon density in Washington, Oregon, and Idaho from 2008 – 2021.

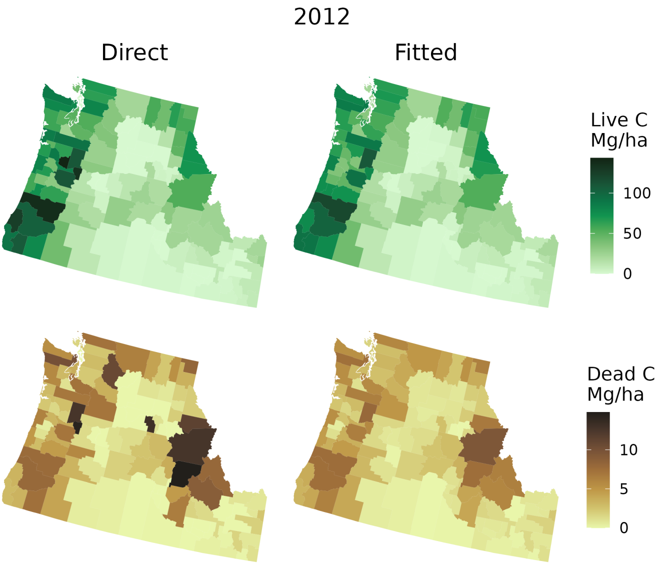
- We do not implement space varying coefficients β_j, but do incorporate dynamically evolving coefficients β_t.

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t \tag{10}$$

$$\boldsymbol{\eta}_t \sim MVN(0, \boldsymbol{\Sigma}_\eta) \tag{11}$$



Preliminary Results



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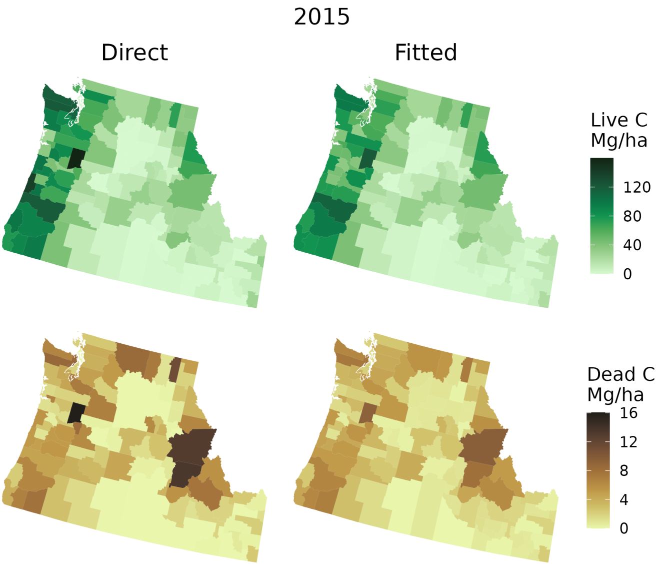

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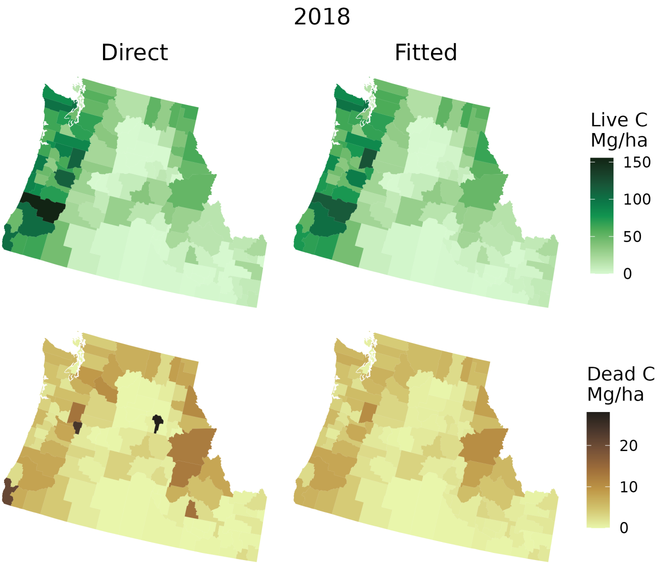

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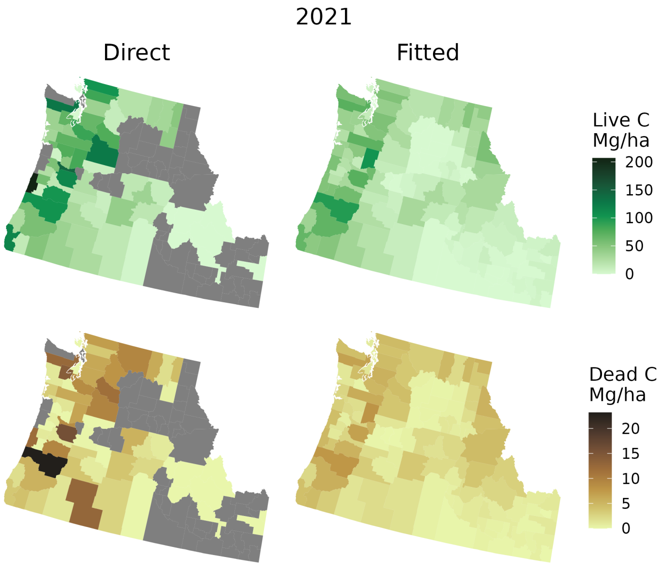

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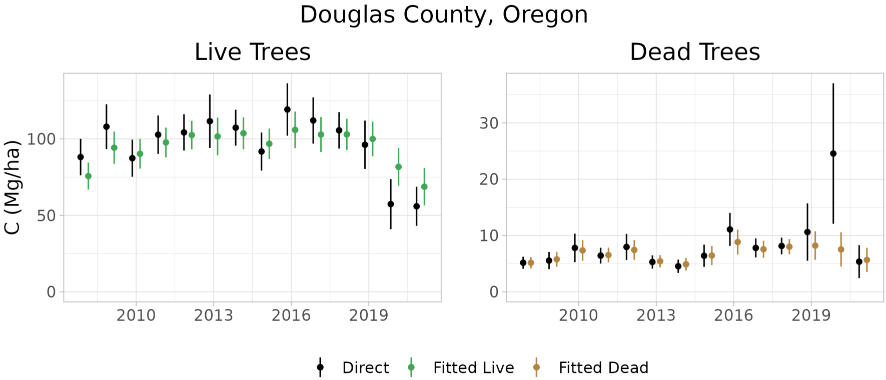
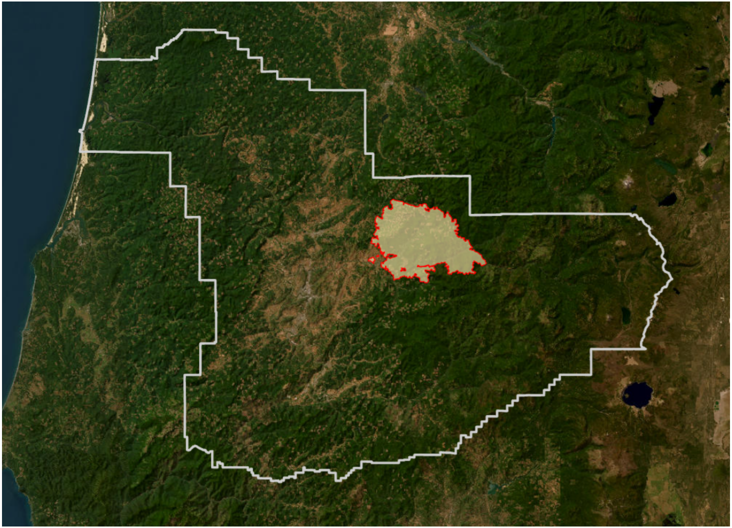


2020 Archie Creek Wildfire



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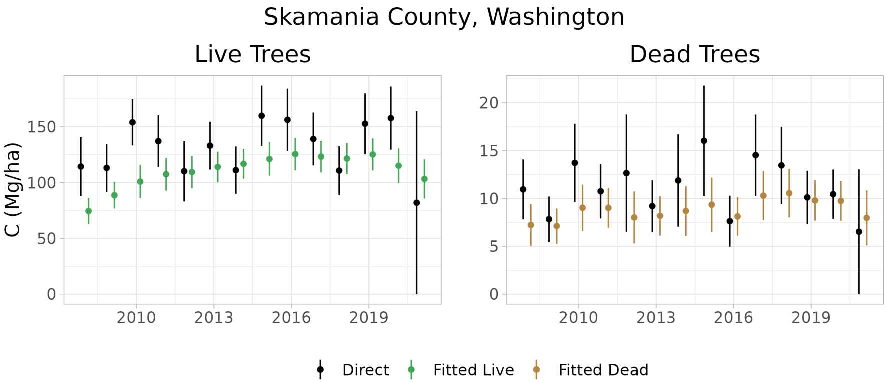

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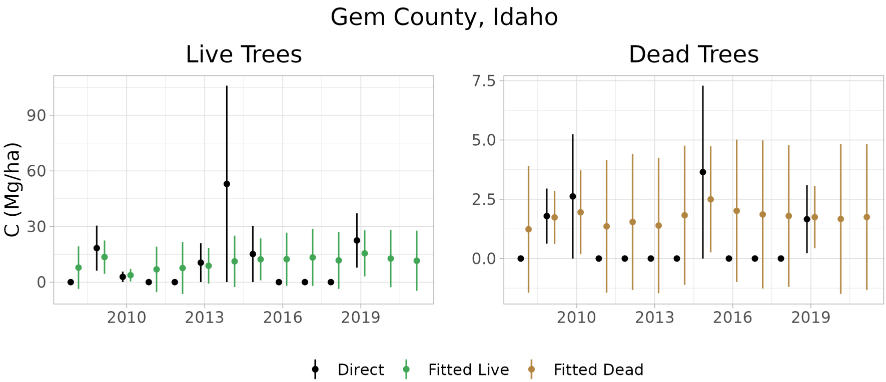
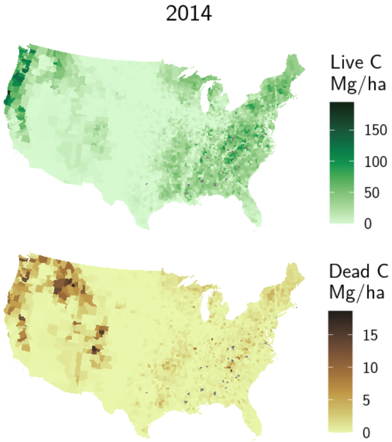


Next Steps

- Extend to entire CONUS
- Incorporate space-varying regression coefficient for TCC.
- Allow $\mathbf{A}$ to vary spatially $\mathbf{A}_j$ and, perhaps, temporally $\mathbf{A}_{j,t}$.
- Other carbon pools (down woody material, leaf litter, soil, etc.)



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