Leveraging national forest inventory data to estimate forest carbon density status and trends for small areas

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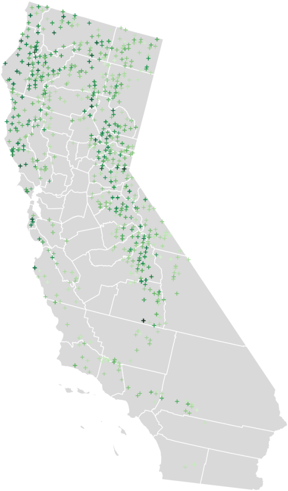
National Forest Inventory Data

- National Forest Inventory (NFI) programs provide critical information on forest health, sustainable management, and ecosystem change.
- Users require higher spatial and temporal resolution forest status and change parameter estimates.
- Design-based estimates are limited to large spatial and temporal scales.

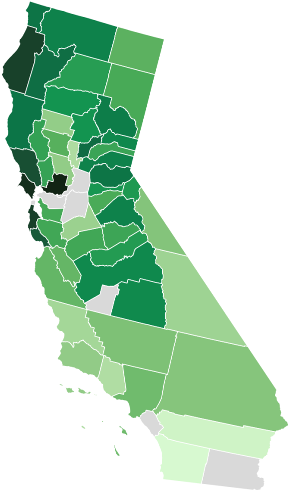


Small Area Estimation

- Small area estimation (SAE) methods have gained attention for estimating forest parameters in data-sparse settings.
  - employ statistical models to relate forest response variables to auxiliary data.
  - improve accuracy and precision over design-based approaches.

Fay-Herriot Model

- The Fay-Herriot (FH) model is widely used in SAE applications for NFI data.
  - fit to small area direct estimates.
  - does not require exact plot locations.
- Direct estimates are often missing when sample sizes are too small or measurements are homogeneous.



Bayesian Spatio-Temporal SAE Model

We propose a Bayesian spatio-temporal SAE model of live forest carbon density (LFCD) that

- directly uses NFI plot-level measurements,
- incorporates auxiliary covariates,
- accommodates spatially and temporally varying regression coefficients,
- appropriately quantifies uncertainty.

Data

- We have 593,368 United States Forest Service Forest Inventory and Analysis (FIA) plot measurements collected across 3,108 counties in the CONUS from 2008 to 2021.
  - Exact plot locations are unknown, but plot measurements may be assigned to counties.
- We leverage remotely sensed percent tree canopy cover (TCC) as a covariate.
  - Averaged among all pixels within given county and year.

Notation

Let

- $j = 1, \ldots, J$ index counties,
- $t = 1, \ldots, T$ index discrete years,
- $i = 1, \ldots, n_{j,t}$ index FIA plots measured in county j in year t,
  - Note, we may have $n_{j,t} = 0$ for some j and t.
- $y_{i,j,t}$ be the LFCD (Mg/ha) at FIA plot i in county j in year t,
- $\mu_{j,t}$ be the latent (unobserved) mean LFCD for county j in year t,
- $\mathbf{x}_{j,t} = (1, x_{1,j,t}, \ldots x_{P,j,t})^{\mathrm{T}}$ be the length $P + 1$ vector of covarites associated with county j in year t,
- $\tilde{\mathbf{x}}_{j,t} = (\tilde{x}_{1,j,t}, \ldots \tilde{x}_{Q,j,t})^{\mathrm{T}}$ be the length $Q \subseteq P$ vector of covariates with space-varying impact on $\mu_{j,t}$.

Model

For county j in year t, the proposed model is then

$$y_{i,j,t} = \underbrace{\mathbf{x}_{j,t}^{\mathrm{T}}\boldsymbol{\beta}_t + \tilde{\mathbf{x}}_{j,t}^{\mathrm{T}}\boldsymbol{\eta}_j + u_{j,t}}_{\mu_{j,t}} + \varepsilon_{i,j,t}, \quad i = 1, \ldots, n_{j,t}, \tag{1}$$

where

- $\varepsilon_{i,j,t} \overset{\text{ind}}{\sim} N(0, \sigma_t^2)$,
- $\boldsymbol{\beta}_t$ is a length $P+1$ vector of temporally-varying regression coefficients,
- $\boldsymbol{\eta}_j$ is a length Q vector of space-varying regression coefficients,
- $u_{j,t}$ is a dynamically evolving spatio-temporal intercept term.

Temporally-varying Regression Coefficients

$\boldsymbol{\beta}_t$ is modeled dynamically as

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\xi}_t, \text{with} \tag{2}$$

$$\boldsymbol{\xi}_t \overset{\text{ind}}{\sim} MVN\left(\mathbf{0}, \boldsymbol{\Sigma}_\xi\right), \quad t = 1, \ldots, T, \tag{3}$$

which allows the effect of covariates in $\mathbf{x}_{j,t}$ to have time-varying impact on the response according to the covariance structure in $\boldsymbol{\Sigma}_\xi$.

Space-varying Regression Coefficients

Writing $\boldsymbol{\eta}_j = (\eta_{1,j}, \ldots, \eta_{Q,j})^{\mathrm{T}}$ and collecting $\boldsymbol{\eta}_q^* = (\eta_{q,1}, \ldots, \eta_{q,J})^{\mathrm{T}}$, we model $\boldsymbol{\eta}_q^*$ as a conditional autoregressive (CAR) random effect,

$$\boldsymbol{\eta}_q^* \sim MVN\left(\mathbf{0}, \tau_{\eta,q}^2 \mathbf{Q}(\rho_{\eta,q})\right), \quad q = 1, \ldots, Q, \tag{4}$$

where

- $\tau_{\eta,q}^2$ is a scalar variance parameter,
- $\rho_{\eta,q}$ is a spatial correlation parameter ($0 < \rho_{\eta,q} < 1$),
- $\mathbf{Q}(\rho_{\eta,q})$ is a $J \times J$ correlation matrix reflecting the county neighborhood structure. (See Banerjee et al. (2004) for details).

Dynamic Spatio-temporal Intercept

$u_{j,t}$ is modeled as a dynamically evolving CAR spatial random effect,

$$u_{j,t} = u_{j,t-1} + \omega_{j,t}, \tag{5}$$

where $u_{j,0} \equiv 0$ for all j.

Then, collecting all $\omega_{j,t}$ for time t as $\boldsymbol{\omega}_t = (\omega_{1,t}, \ldots, \omega_{J,t})^{\mathrm{T}}$, we specify a CAR spatial structure for $\boldsymbol{\omega}_t$ as

$$\boldsymbol{\omega}_t \sim MVN\left(\mathbf{0}, \tau^2_{\omega,t}\mathbf{Q}(\rho_\omega)\right). \tag{6}$$

Direct Estimates

- Traditionally, NFI-derived quantities of interest have been estimated using design-based direct estimates.

Specifically, the direct estimate mean for $\mu_{j,t}$ is calculated as

$$\hat{\mu}_{j,t} = \frac{1}{n_{j,t}} \sum_{i=1}^{n_j t} y_{i,j,t}, \tag{7}$$

with associated estimate variance

$$\hat{\sigma}^2_{j,t} = \frac{1}{n_{j,t}(n_{j,t} - 1)} \sum_{i=1}^{n_j t} (y_{i,j,t} - \hat{\mu}_{j,t})^2. \tag{8}$$

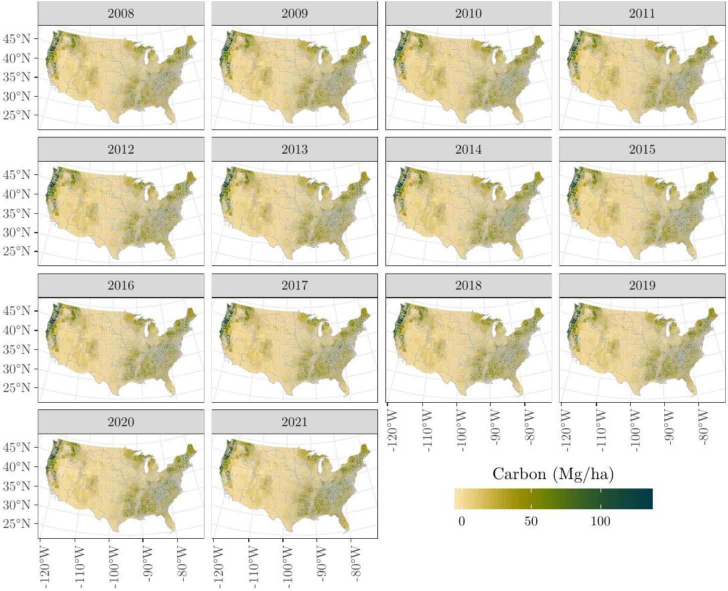


Figure 1: Posterior mean values of live forest carbon density ($\mu_{j,t}$).

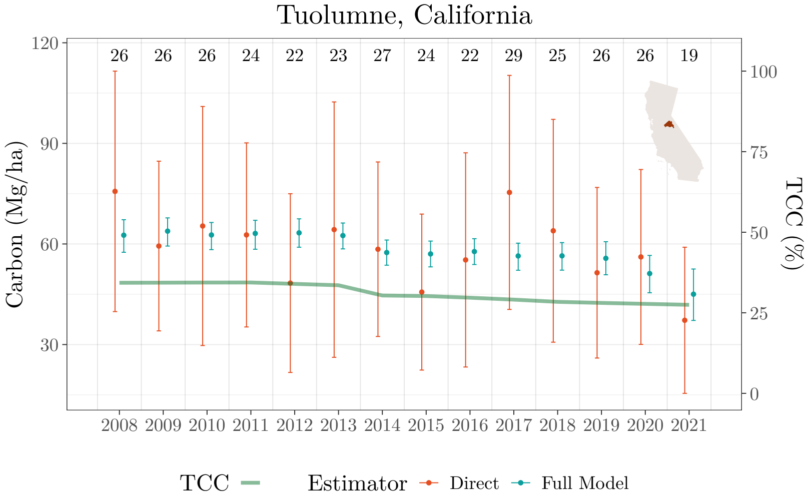


Figure 2: Posterior mean and 95% credible intervals of LFCD ($\mu_{j,t}$) for Tuolumne County, California, compared to direct estimate means ($\hat{\mu}_{j,t}$) and 95% confidence intervals over time. Top row displays sample sizes ($n_{j,t}$).

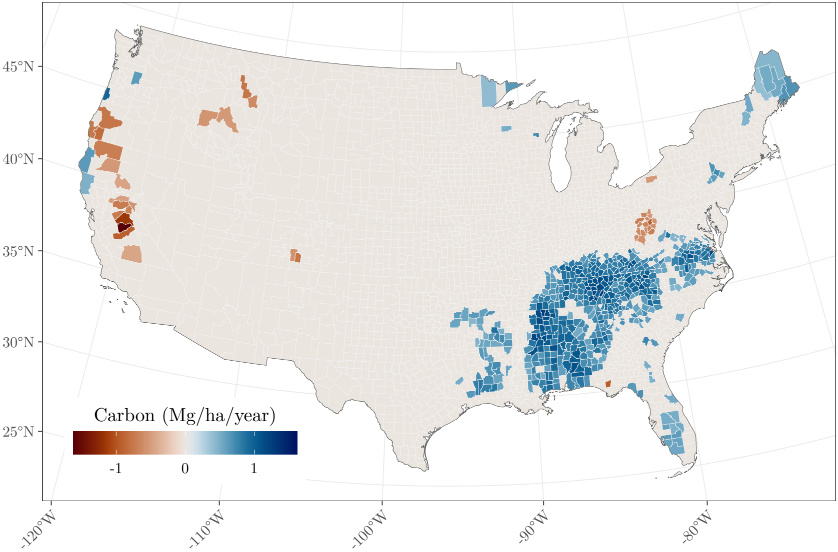


Figure 3: Significant live forest carbon density trends (Mg/ha/year).

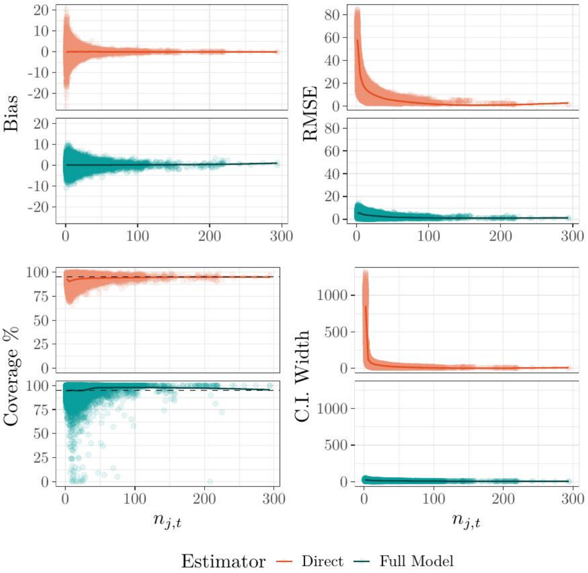


Figure 4: Average measures of bias, root mean square error (RMSE), coverage percentage, and coverage interval widths for the model and direct estimator arranged according to sample size $n_{j,t}$.

Each point represents the mean metric value for estimating $\mu_{j,t}$ averaged over $R = 100$ simulated population replicates.

Thank you

Shannon et al. (2025)

References

Banerjee, S., Carlin, B., and Gelfand, A. (2004). *Hierarchical Modeling and Analysis of Spatial Data*, volume 101.

Shannon, E. S., Finley, A. O., May, P. B., Domke, G. M., Andersen, H.-E., III, G. C. G., Nothdurft, A., and Banerjee, S. (2025). Leveraging national forest inventory data to estimate forest carbon density status and trends for small areas.